Bengt Muthen, January 2013

It is of interest to understand if factor loading invariance can be tested separately from threshold invariance. In the case of continuous outcomes, invariance of factor loadings make it possible to identify and estimate factor variance-covariances in the different groups, while intercept invariance is necessary only for identifying and estimating factor means in the different groups. This holds even if there is residual variance noninvariance because the residual variances do not influence the conditional expectation function of the outcome given the factors. In contrast, with binary outcomes, the residual variances do influence the conditional expectation function, that is, the item characteristic curve. In the binary case, a model with non-invariant thresholds is not identified when allowing group-varying residual variances. To see the indeterminacies, consider multiplying all scale factors by the same constant in a certain group. This change can be absorbed in the factor variance and in the thresholds. This implies that threshold invariance and factor loading invariance cannot be separately tested in the binary case without further restrictions, one case being residual variance invariance (see also Millsap & Tien, 2004 and Millsap, 2011). Muthen and Asparouhov (2002) discuss further identification and testing matters for multiple-group analysis and show the equivalent issues for invariance across time in growth models.

In the polytomous case, each item has more than one threshold and the identification status is different from the binary case. Millsap and Tien (2004) and Millsap (2011) give identification rules for invariance restrictions on model parameters. As these authors show, it is possible to identify non-invariant factor loadings in conjunction with a minimal set of restrictions on the thresholds, while at the same time allowing group-varying factor means, factor variances, and residual variances. As in the binary case, a reference group fixes the factor mean to zero and the residual variances to one. For the single-factor case, the minimal set of restrictions on the thresholds requires (i) one threshold per item to be held equal across groups and (ii) one more threshold to be held equal across groups for the item that sets the factor metric. Restriction (i) eliminates indeterminacies involving scale factors, factor loadings, and factor variances. Restriction (ii) eliminates indeterminacies involving scale factors, thresholds, and factor means. Different choices of metric-setting items produce the same model fit. The authors refer to this model as the baseline model.